Direct Method for Finding an Optimal Solution for Fuzzy Transportation Problem

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ABSTRACT

In this paper we shall study fuzzy transportation problem, and we introduce an approach for solving a wide range of such problem by using a method which apply it for ranking of the fuzzy numbers. Some of the quantities in a fuzzy transportation problem may be fuzzy or crisp quantities. In many fuzzy decision problems, the quantities are represented in terms of fuzzy numbers may be triangular or trapezoidal. Thus, some fuzzy numbers are not directly comparable. First, we transform the fuzzy quantities as the cost, coefficients, supply and demands, in to crisp quantities by using Robust's ranking method [1] and then by using the classical algorithms we solve and obtain the solution of the problem. The new method is a systematic procedure, easy to apply and can be utilized for all types of transportation problem whether maximize or minimize objective function. At the end, this method is illustrated with a numerical example.

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Keywords: Fuzzy ranking, fuzzy sets (normal and convex), Membership Functions, Trapezoidal fuzzy number, Triangular fuzzy number, Optimal Solution, Transportation problem.

I. INTRODUCTION

The transportation problem is a special linear programming problem which arises in many practical applications. In this problem we determine optimal shipping patterns between origins or sources and destinations. Suppose that m origins are to supply n destinations with a certain product. Let a_i be the amount of the product available at origin i, and b_j be the amount of the product required at destination j. Further, we assume that the cost of shipping a unit amount of the product from origin i to destination j is c_{ij}, we then let x_{ij} represent the amount shipped from origin i to destination j. If shipping costs, are assumed to be proportional to the amount shipped from each origin to each origin to each destination so as to minimize total shipping cost turns out be a linear programming problem. Transportation models have wide applications in logistics and supply chin for reducing the cost. When the cost coefficients and the supply and demand quantities are known exactly. A fuzzy transportation problem is a transportation problem in which the transportation cost, supply and demand quantities are fuzzy quantities.

In many fuzzy decision problems, the data are represented in terms of fuzzy numbers. In a fuzzy transportation problem, all parameters are fuzzy numbers. Fuzzy numbers may be triangular or trapezoidal. Thus, some fuzzy numbers are not directly comparable. Comparing between two or multi fuzzy numbers and ranking such a numbers is one of the import subjects, and how to set the rank of fuzzy numbers has been one of the main problems. Several methods are introduced for ranking of fuzzy numbers. Here we use Robut's ranking method [1] which satisfies the properties of compensation, linearity and additivity. This method is very easy to understand and apply.

II. PRELIMINARIES

1.1 Fuzzy Set:

A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval [0, 1].(i.e.) $A=\{(x,\mu_A(x));x\in X\}$. Here μ_A : X →[0,1] is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x∈ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

1.2 Normal fuzzy set:

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ∈ X such that $\mu_A(x)$=1.

1.3 Convex:

A fuzzy set A is convex if and only if, for any $x_1, x_2 \in X$, the membership function of A satisfies the

inequality $\mu_A(\lambda x_1+(1-\lambda x_2))\geq \min(\mu_A(x_1),\mu_A(x_2)),0\leq\lambda\leq 1.$

1.4 Triangular Fuzzy Number:

For a triangular fuzzy number A(x), it can be represented by A (a,b,c;1) with membership function μ(x) given by

$$\mu(x)=\begin{cases}\dfrac{x-a}{b-a}, & a\leq x\leq b\\ 1\quad , & x=b\\ \dfrac{c-x}{c-b}, & \mathrm{b}\leq x\leq c\\ 0 & ,otherwise\end{cases}$$

1.5 Trapezoidal fuzzy number:

For a trapezoidal fuzzy number A(x), it can be represented by A (a, b, c, d; 1) with membership function μ(x) given by

$$\mu(x)=\begin{cases}\dfrac{x-a}{b-a}, & a\leq x\leq b\\ 1\quad , & \mathrm{b}\leq x\leq c\\ \dfrac{d-x}{d-c}, & \mathrm{c}\leq x\leq d\\ 0 & ,otherwise\end{cases}$$

1.6 α-Cut:

The α-cut of a fuzzy number $A(x)$ is defined as $A(\alpha)=\{x/\mu(x)\geq\alpha,\alpha\in[0,1]\}$.

1.7 Arithmetic operations between two triangular and trapezoidal fuzzy numbers fuzzy numbers:

Addition and Subtraction of two triangular fuzzy numbers can be performed as

$$\tilde{A}+\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3)$$

$$\tilde{A}-\tilde{B}=(a_1-b_1,a_2-b_2,a_3-b_3)$$

Addition and Subtraction of two trapezoidal fuzzy numbers can be performed as

$$\tilde{A}+\tilde{B}=\left(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4,\right)$$
$$\tilde{A}-\tilde{B}=\left(a_1-b_1,a_2-b_2,a_3-b_3,a_4-b_4\right)$$

III. ROBUST'S RANKING TECHNIQUE [1]

Robust's ranking technique[1] which satisfy compensation, linearity, and additively properties and provides results which are consistent with human intuition. If ã is a fuzzy number then the Robust's ranking is defined by

$$R(\tilde{a})=\int_0^1 0.5\left(a_\alpha^l,a_\alpha^u\right)d\alpha, \text{Where} \left(a_\alpha^l,a_\alpha^u\right) \text{is the } \alpha\text{- level cut of the fuzzy number } \tilde{a}.$$

In this paper we use this method for ranking the objective values. The Robust's ranking index R(ã) gives the representative value of fuzzy number ã. It satisfies the linearity and additive property.

IV. FUZZY TRANSPORTATION MODEL FORMULATION

We deal with the production and transportation planning of a certain manufacturer that has production facilities and central stores for resellers in several sites in Chennai. Each store can receive products from all production plants and it is not necessary that all products are produced in all production units.

Assume that a logistics center seeks to determine the transportation plan of a homogeneous commodity from m sources to n destinations. Each source has an available supply of the commodity to distribute to various destinations, and each destination has a forecast demand of the commodity to be received from various sources. This work focuses on developing a Fuzzy linear programming method for optimizing the transportation plan in fuzzy environments.

1.8 **Index sets**

i index for source, for all $i = 1, 2, \ldots, m$

j index for destination, for all $j = 1, 2, \ldots, n$

g index for objectives, for all $g = 1, 2, \ldots, k$

1.9 **Decision variables**

x_{ij} units transported from source i to destination j (units)

1.10 **Objective functions**

$\tilde{Z}$ Transportation costs (Rs.)

1.11 **Parameters**

$\tilde{C}_{ij}$ Transportation cost per unit delivered from source i to destination j (Rs/unit)

$\tilde{S}_i$ Total available supply at each source i (units)

$\tilde{D}_j$ Total forecast demand at each destination j (units)

1.12 **Objective functions**

Minimize total transportation costs

$$Min\ \tilde{Z}=\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$$

Constraints on total available supply for each source i

$$\sum_{i=1}^{n}x_{ij}=\tilde{S}_j$$

Constraints on total forecast demand for each destination j

$$\sum_{j=1}^{m}x_{ij}=\tilde{D}_i$$

If any of the parameters x_{ij}, S_i, and D_j is fuzzy, the total transportation cost Z becomes fuzzy as well. The conventional transportation problem defined then turns into the fuzzy transportation problem.

V. FUZZY TRANSPORTATION MODEL ILLUSTRATION

Consider transportation with *m* fuzzy origins (rows) and *n* fuzzy destinations (columns). Let $\tilde{C}_{ij} = \left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right]$ be the cost of transporting one unit of the product from i^{th} fuzzy origin to j^{th} fuzzy destination. $\tilde{S}_i = \left[s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}\right]$ be the quantity of commodity available at fuzzy origin i , $\tilde{D}_j = \left[d_j^{(1)}, d_j^{(2)}, d_j^{(3)}, d_j^{(4)}\right]$ the quantity of commodity needed at fuzzy destination j. X_{ij} is quantity transported from i^{th} fuzzy origin to j^{th} fuzzy destination. The above fuzzy transportation problem can be stated in the below tabular form.

	1	2		n	Fuzzy Supply
1	$\tilde{c}_{11}x_{11}$	$\tilde{c}_{12}x_{12}$	…	$\tilde{c}_{1n}x_{1n}$	$\tilde{S}_1$
2	$\tilde{c}_{21}x_{21}$	$\tilde{c}_{22}x_{22}$	… ….	$\tilde{c}_{2n}x_{2n}$	$\tilde{S}_2$
m	$\tilde{c}_{m1}x_{m1}$	$\tilde{c}_{m2}x_{m2}$	…. ….	$\tilde{c}_{mn}x_{mn}$	$\tilde{S}_m$
Fuzzy Demand	$\tilde{D}_1$	$\tilde{D}_2$		$\tilde{D}_n$	$\sum_{j=1}^{n}\tilde{D}_j = \sum_{i=1}^{m}\tilde{S}_i$

Where $\tilde{C}_{ij} = \left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right]$

$$\tilde{X}_{ij} = \left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right]$$

$$\tilde{S}_i = \left[s_i^{(1)}, s_i^{(2)}, s_i^{(3)}, s_i^{(4)}\right]$$

$$\tilde{D}_j = \left[d_j^{(1)}, d_j^{(2)}, d_j^{(3)}, d_j^{(4)}\right]$$

VI. METHODOLOGY

Step 1: Construct the transportation table from given fuzzy transportation problem.
Step 2: Subtract each row entries of the transportation table from the respective row minimum and then subtract each column entries of the resulting transportation table from respective column minimum.
Step 3: Now there will be at least one zero in each row and in each column in the reduced cost matrix. Select the first zero (row-wise) occurring in the cost matrix. Suppose $(i, j)^{th}$ zero is selected, count the total number of zeros (excluding the selected one) in the i^{th} row and j th column. Now select the next zero and count the total number of zeros in the corresponding row and column in the same manner. Continue it for all zeros in the cost matrix.

Step 4: Now choose a zero for which the number of zeros counted in step 3 is minimum and supply maximum possible amount to that cell. If tie occurs for some zeros in step 3 then choose a $(k,l)^{th}$ zero breaking tie such that the total sum of all the elements in the k^{th} row and l^{st} column is maximum. Allocate maximum possible amount to that cell.
Step 5: After performing step 4, delete the row or column for further calculation where the supply from a given source is depleted or the demand for a given destination is satisfied.
Step 6: Check whether the resultant matrix possesses at least one zero in each row and in each column. If not ,repeat step 2, otherwise go to step 7.
Step 7: Repeat step 3 to step 6 until and unless all the demands are satisfied and all the supplies are exhausted.

VII. NUMERICAL EXAMPLE

Example 5.1 Consider the following fuzzy transportation problem.

A company has four sources $S_{1,}S_{2,}S_{3,}$ *and* S_4 and four destinations $D_{1,}D_{2,}D_{3,}$ *and* D_4 the fuzzy transportation cost for unit quantity of the product form i[th] source to j[th] destination is C_{ij} where

$$[\tilde{C}_{ij}]_{4X4} = \begin{pmatrix} (5,10,15) & (5,10,20) & (5,15,20) & (5,10,15) \\ (5,10,20) & (5,15,20) & (5,10,15) & (10,15,20) \\ (5,10,20) & (10,15,20) & (10,15,20) & (5,10,15) \\ (10,15,25) & (5,10,15) & (10,20,30) & (10,15,25) \end{pmatrix}$$

and fuzzy availability of the product at source are (10,15,20),(5,10,15),(20,30,40),(15,20,25)) and the fuzzy demand of the product at destinations are(25,30,35),(10,15,20),(5,15,20),

(10,15,25)) respectively. The fuzzy transportation problems are

$$\begin{array}{cccccc} & D_1 & D_2 & D_3 & D_4 & FUZZY\ CAPACITY \\ S_1 & (5,10,15) & (5,10,20) & (5,15,20) & (5,10,15) & (10,15,20) \\ S_2 & (5,10,20) & (5,15,20) & (5,10,15) & (10,15,20) & (5,10,15) \\ S_3 & (5,10,20) & (10,15,20) & (10,15,20) & (5,10,15) & (20,30,40) \\ S_4 & (10,15,25) & (5,10,15) & (10,20,30) & (10,15,25) & (15,20,25) \\ \text{FUZZY DEMAND} & (25,30,35) & (10,15,20) & (5,15,20) & (10,15,25) & \end{array}$$

Solution:

Step 1: Construct the transportation table from given fuzzy transportation problem.

$$\begin{aligned} MinZ = {} & R(5,10,15)x_{11} + R(5,10,20)x_{12} + R(5,15,20)x_{13} + R(5,10,15)x_{14} \\ & R(5,10,20)x_{21} + R(5,15,20)x_{22} + R(5,10,15)x_{23} + R(10,15,20)x_{24} \\ & R(5,10,20)x_{31} + R(10,15,20)x_{32} + R(10,15,20)x_{33} + R(5,10,15)x_{34} \\ & R(10,15,25)x_{41} + R(5,10,15)x_{42} + R(10,20,30)x_{43} + R(10,15,25)x_{44} \end{aligned}$$

Now we calculate R(5,10,15) by applying Robst's ranking method. The membership function of the triangular fuzzy number (5,10,15) is

$$\mu(x) = \begin{cases} \dfrac{x-5}{5}, & 5 \le x \le 10 \\ 1 \quad , & x = 10 \\ \dfrac{15-x}{5}, & 10 \le x \le 15 \\ 0 & ,otherwise \end{cases}$$

The α-Cut of the fuzzy number (5,10,15) is $(a_\alpha^l, a_\alpha^u) = (5\alpha + 5, 15 - 5\alpha)$ for which $R(a_{1,1}) = R(5,10,15) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u)d\alpha = \int_0^1 0.5(20)d\alpha = 10$

Proceeding similarly, the Robust's ranking indices for the fuzzy costs a_{ij} are calculated as:

$R(a_{1,2})=11.25,\ R(a_{1,3})=13.75,\ R(a_{1,4})=10,\ R(a_{2,1})=11.25,\ R(a_{2,2})=13.75,$

$R(a_{2,3})=10,\ R(a_{2,4})=15,\ R(a_{3,1})=11.25,\ R(a_{3,2})=15,\ R(a_{3,3})=15,$

$R(a_{3,4})=10,\ R(a_{4,1})=16.25,\ R(a_{4,2})=10,\ R(a_{4,3})=20\ ,\ R(a_{4,4})=16.25$

Rank of All Supply $R(10,15,20)=15\ ,\ R(5,10,15)=10,\ R(20,30,40)=30,\ R(15,20,25)=20$

Rank of All Demand $R(25,30,35)=30\ ,\ R(10,15,20)=15,(5,15,20)=13.75,\ R(10,15,25)=16.25$

We replace these values for their corresponding $\tilde{a}_{ij}$ in which result in a convenient transportation problem is.

$$\begin{array}{cc} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \quad CAPACITY \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} & \begin{pmatrix} 10 & 11.25 & 13.75 & 10 \\ 11.25 & 13.75 & 10 & 15 \\ 11.25 & 15 & 15 & 10 \\ 16.25 & 10 & 20 & 16.25 \end{pmatrix} \begin{array}{c} 15 \\ 10 \\ 30 \\ 20 \end{array} \\ \text{DEMAND} & \begin{array}{cccc} 30 & 15 & 13.75 & 16.25 \end{array} \end{array}$$

Step 2: Subtract each row entries of the transportation table from the respective row minimum and then subtract each column entries of the resulting transportation table from respective column minimum.

$$\begin{array}{cc} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \quad CAPACITY \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} & \begin{pmatrix} 0 & 1.25 & 3.75 & 0 \\ 1.25 & 3.75 & 0 & 5 \\ 1.25 & 5 & 5 & 0 \\ 6.25 & 0 & 0 & 6.25 \end{pmatrix} \begin{array}{c} 15 \\ 10 \\ 30 \\ 20 \end{array} \\ \text{DEMAND} & \begin{array}{cccc} 30 & 15 & 13.75 & 16.25 \end{array} \end{array}$$

Step 3: Now there will be at least one zero in each row and in each column in the reduced cost matrix. Select the first zero (row-wise) occurring in the cost matrix. Suppose (i, j)th zero is selected, count the total number of zeros (excluding the selected one) in the ith row and j th column. Now select the next zero and count the total number of zeros in the corresponding row and column in the same manner. Continue it for all zeros in the cost matrix.

$$\begin{array}{cc} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \quad CAPACITY \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} & \begin{pmatrix} 0 & 1.25 & 3.75 & 0 \\ 1.25 & 3.75 & 0 & 5 \\ 1.25 & 5 & 5 & 0 \\ 6.25 & 0 & 10 & 6.25 \end{pmatrix} \begin{array}{cc} 15 & (2) \\ 10 & (1) \\ 30 & (1) \\ 20 & (1) \end{array} \\ \text{DEMAND} & \begin{array}{cccc} 30 & 15 & 13.75 & 16.25 \\ (1) & (1) & (1) & (1) \end{array} \end{array}$$

Step 4: Now choose a zero for which the number of zeros counted in step 3 is minimum and supply maximum possible amount to that cell. If tie occurs for some zeros in step 3 then choose a (k,l)th zero breaking tie such that the total sum of all the elements in the kth row and lst column is maximum. Allocate maximum possible amount to that cell.

$$\begin{array}{cc} & \begin{array}{ccccc} D_1 & D_2 & D_3 & D_4 & CAPACITY \end{array} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} & \left(\begin{array}{cccc} 0(15) & 1.25 & 3.75 & 0 \\ 1.25 & 3.75 & 0 & 5 \\ 1.25 & 5 & 5 & 0 \\ 6.25 & 0 & 10 & 6.25 \end{array}\right)\begin{array}{c} 0 \\ 10 \\ 30 \\ 20 \end{array} \\ \text{DEMAND} & \begin{array}{cccc} 15 & 15 & 13.75 & 16.25 \end{array} \end{array}$$

Step 5: After performing step 4, delete the row or column for further calculation where the supply from a given source is depleted or the demand for a given destination is satisfied.

$$\begin{array}{cc} & \begin{array}{ccccc} D_1 & D_2 & D_3 & D_4 & CAPACITY \end{array} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} & \left(\begin{array}{cccc} 0(15) & 1.25 & 3.75 & 0 \\ 1.25 & 3.75 & 0 & 5 \\ 1.25 & 5 & 5 & 0 \\ 6.25 & 0 & 10 & 6.25 \end{array}\right)\begin{array}{c} 0 \\ 10 \\ 30 \\ 20 \end{array} \\ \text{DEMAND} & \begin{array}{cccc} 15 & 15 & 13.75 & 16.25 \end{array} \end{array}$$

Step 6: Check whether the resultant matrix possesses at least one zero in each row and in each column. If not, repeat step 2, otherwise go to step 7.
Step 7: Repeat step 3 to step 6 until and unless all the demands are satisfied and all the supplies are exhausted.

$$\begin{array}{cc} & \begin{array}{ccccc} D_1 & D_2 & D_3 & D_4 & CAPACITY \end{array} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} & \left(\begin{array}{cccc} 10(15) & 11.25 & 13.75 & 10 \\ 11.25 & 13.75 & 10(10) & 15 \\ 11.25(10) & 15 & 15(3.75) & 10(16.25) \\ 16.25(5) & 10(15) & 20 & 16.25 \end{array}\right)\begin{array}{c} 15 \\ 10 \\ 30 \\ 20 \end{array} \\ \text{DEMAND} & \begin{array}{cccc} 30 & 15 & 13.75 & 16.25 \end{array} \end{array}$$

The total cost associated with the allocation is 812.5

VIII. CONCLUSIONS

The Direct method provides an optimal solution directly, in less iteration, for the transportation problems. As this method consumes less time and is very easy to understand and apply, so it will be very helpful for decision makers who are dealing with logistic and supply chain problems.

REFERENCES

[1]. A.Solairaju and R.Nagarajan "Computing Improved Fuzzy Optimal Hungarian Assignment Problems with Fuzzy Costs under Robust Ranking Techniques"(2010)

[2]. A.Srinivasan and G.Geetharamani" Method for solving fuzzy assignment problem using ones assignment method and Robust's ranking technique" Applied MathematicalSciences,Vol.7,2013,no.113,5607-5619.

[3]. A.ZADEH "Fuzzy Set"(1965)

[4]. AbdulQuddos,ShakeelJavaid,M.M Khalid " A New Method for Finding an Optimal Solution for Transportation Problems" International Journal on Computer Science and Engineering (IJCSE), ISSN : 0975-3397 ,Vol. 4 No. 07 July 2012.

[5]. George J.Klir/Bo Yoan "Fuzzy Sets and Fuzzy logic Theory and Applications"

[6]. H.J Zimmermann, "Fuzzy set theory and its Applications", third Ed.,kluwar Academic,Boston1996.

[7]. Hamdy A.Taha,"Operations Research", an introduction 8th Ed.(2007)

[8]. JieLu.Guangquan Zhang and DaRuan.Fengjie Wu "Multi objective Group Decision Making Methods, Software and Applications with Fuzzy Set Techniques".

[9]. Kadhirvel.K,Balamurugan.K, "Method For Solving Hungarian Assignment Problems Using Triangular And Trapezoidal Fuzzy Number" (2012)

[10]. M.Hellmann "Fuzzy logic Introduction"

[11]. R.R Yager,"A procedure for ordering fuzzy subsets of the unit interval, information Sciences",24,(1982),143-161

[12]. Y. L. P. Thorani, P. PhaniBushanRao, and N. Ravi Shankar "Ordering Generalized Trapezoidal Fuzzy Numbers Using Orthocenter of Centroids"